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Numerical study of the transition of the four-dimensional random field Ising model

Roberto Sacconi

Dipartimento di Fisica, Università di Roma, 'La Sapienza', Piazzale A. Moro 2, 00185 Roma, Italy

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Abstract. We study numerically the region above the critical temperature of the four-dimensional random field Ising model. Using a cluster dynamic we measure the connected and disconnected magnetic susceptibility and the connected and disconnected overlap susceptibility. We use a bimodal distribution of the field with $h_R = 0.35T$ for all temperatures and a lattice size $L = 16$. Through a least-square fit we determine the critical temperature at which the two susceptibilities diverge. We also determine the critical exponents γ and $\bar{\gamma}$. We find that the magnetic susceptibility and the overlap susceptibility diverge at two different temperatures. This is coherent with the existence of a glassy phase above T_c. Accordingly with other simulations we find $\bar{\gamma} = 2\gamma$. In this case we have a scaling theory with two independent critical exponents.

1. Introduction

In the last few years the random field Ising model [1], or RFIM, has attracted a lot of attention. Despite great efforts the critical behaviour of the model is still not clear. Both numerical and analytical studies have shown that in three dimensions at low temperature and sufficiently small field strength there is a transition from a disordered phase to a long-range ordered phase. This result was first suggested by Imry and Ma [2]. They considered the possibility that the model could be split in clusters of dimension L. Through a direct comparison between ferromagnetic and random field energy they found a value of two for the lower critical dimension d_l. Subsequent arguments [3], based on perturbative expansion led to the result that the critical behaviour of the RFIM should be equivalent to that of the Ising model in two fewer dimensions. This suggested a dimensional reduction of two so that, if this should be taken as a general rule, the lower critical dimension should be three instead of two. However, there is a rigorous proof, see Imbrie [4], that the lower critical dimension is two. It can be shown that, for a certain range of temperatures, the mean-field equation has more than one solution; this is related to the fact that this model has a complex free energy landscape. This is essentially the reason why the dimensional reduction fails. An accurate numerical investigation of the mean-field equation has been done by Guagnelli *et al* [13] and successively by Lancaster *et al* [12]. They found that the mean-field equation has more than one solution when the *correlation length* is still finite. In *spin glass* [11] mean field theory we have a similar situation. It seems reasonable to use, in this case too, replica symmetry breaking theory (RSB) such as that used by Parisi in that context. Mezárd *et al* [8, 9], using RSB techniques and the self-consistent screening approximation (SCSA) [10], have shown the existence of a region above the critical temperature in which there should be

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a 'glassy' phase. In this case we have two different values of the critical temperature: one called T_c, which is the usual critical temperature of a ferromagnetic system, and another, called T_b so that $T_b > T_c$, at which we have a transition from a paramagnetic phase to a 'glassy' phase. In section 2 we first discuss the scaling theory of the model and then we introduce the concept of replica susceptibility. In section 3 we give a brief description of the algorithm used and then we report our results and conclusions in the last two sections.

2. Theory

The RFIM is defined by the Hamiltonian

$$H_{\mathrm{RFIM}} = -J \sum_{\langle i,j \rangle} \sigma_i \sigma_j - \sum_i h_i \sigma_i. \tag{1}$$

The variables σ_i are Ising-like spins and h_i are independent random variables with mean $\langle h_i \rangle_{\mathrm{av}} = 0$ and variance $\langle h_i^2 \rangle_{\mathrm{av}} = h_R^2$. The typical distribution used is the Gaussian or bimodal distribution.

We first discuss the prediction of the scaling theory. Bray and Moore and independently, Fisher [5, 6] proposed a scaling theory based on the assumption of a second-order phase transition with a zero-temperature fixed point. At $T = 0$ for the correlation length, as usual, we expect a power-law behaviour given by

$$\xi \propto t^{-\nu}.$$

In this case t is such that

$$t = \frac{h_R}{J} - \left(\frac{h_R}{J}\right)^*$$

where $(h_R/J)^*$ is the value of h_R/J at the fixed point. The other relevant parameters are

$$h \qquad J$$

where h is a uniform external field. Because for the RFIM the coupling constant is not fixed this yields a change of the energy scale. In this case we obtain the modified hyperscaling relation

$$(2 - \alpha) = (d - y)\nu$$

where y is the critical exponent related to J. If b is the length scale factor of the renormalization group coarse-graining transformation we have

$$J' = b^y J.$$

Another difference with the Ising model is related to the correlation function. The presence of means over the quenched field causes the correlation function to have two different types of behaviour. We have a connected and a disconnected correlation function

$$G_{\mathrm{con}}(r) \equiv \overline{\langle \sigma_0 \sigma_r \rangle - \langle \sigma_0 \rangle \langle \sigma_r \rangle} = \frac{T}{r^{d-2+\eta}} g(r/\xi) \tag{2}$$

$$G_{\mathrm{dis}}(r) \equiv \overline{\langle \sigma_0 \rangle \langle \sigma_r \rangle} = \frac{T}{r^{d-4+\bar{\eta}}} g'(r/\xi). \tag{3}$$

This defines another critical exponent $\bar{\eta}$. The $\langle \ldots \rangle$ and the $\overline{(\ldots)}$ denotes, respectively, the thermal average and the average over different random field configurations. The other scaling relations are still valid in this case

$$\alpha + 2\beta + \gamma = 2 \qquad \delta = \Delta/\beta \ldots.$$

In summary, we have eleven critical exponents and eight scaling relations. There seems to be a phase transition with three independent exponents. Schwartz and Soffer [7] have demonstrated the inequality

$$\bar{\eta} \leqslant 2\eta. \tag{4}$$

Numerical simulations [15] have suggested that (4) should be fulfilled like an equality. In this case we return to a two independent exponents transition.

If the transition is a glass-like-spin transition then the correct order parameter of the theory is the *overlap*

$$q = \frac{1}{N}\sum_{i=1}^{N}\sigma_i\tau_i \tag{5}$$

where σ_i and τ_i are two generic spins of the two replica system.

In this case we are not interested in the magnetization correlation function but in the *replica correlation function*. In more detail we can define a magnetic susceptibility and an overlap susceptibility

$$\begin{aligned}
\overline{\chi(m)_{\mathrm{con}}} &= N[\overline{\langle m^2\rangle} - \overline{\langle m\rangle^2}] \\
\overline{\chi(m)_{\mathrm{dis}}} &= N[\overline{\langle m\rangle^2}] \\
\overline{\chi(q)_{\mathrm{con}}} &= N[\overline{\langle q^2\rangle} - \overline{\langle q\rangle^2}] \\
\overline{\chi(q)_{\mathrm{dis}}} &= N[\overline{\langle q\rangle^2}]
\end{aligned} \tag{6}$$

where N is the volume of the lattice and we then have a distinction between the connected and the disconnected parts. In this paper, by means of the Monte Carlo simulation, we measure the four quantities in (6) in the region slightly above the critical temperature. In this way we are able to make a comparison between the susceptibility related to the magnetization and that related to the overlap. If the three temperature transition scheme proposed in [9] is correct then the overlap and the magnetic susceptibility must diverge at two different temperatures.

3. The algorithm

The algorithm used to carry out the simulations is a generalization of the cluster algorithm proposed by Wolff [17] for the Ising model. According to the limited cluster flipped algorithm proposed by Newmann and Barkema [16] we have realized an algorithm capable of flipping more than one spin at a time. The algorithm is capable of forming clusters with limited size. A Monte Carlo step consists of the following two points:

(i) Build a cluster.

• We choose a random site of the lattice. Then we choose, according to a certain distribution probability, the maximum size, R, of the cluster. As is explained in [16] the appropriate choice of this probability distribution is of fundamental importance. In this work we use a power-law distribution with $P(R) = 1/R^{\alpha}$ and $\alpha = 2$.

• We add similarly oriented neighbouring spins. If the spin under consideration is within the allowed distance then we add it with probability $1 - \exp(-2\beta J)$.

• We repeat the above step until there are no more spins to add to the cluster.

(ii) Once the cluster is created we attempt to flip the spins inside it.

The cluster will be flipped with a probability factor proportional to the random field and to the number of spins s which might have been added but which are found just outside the

radius of the cluster. In detail we have

$$\begin{cases} P_{\text{flip}} = \exp(-2\beta J s) & \text{if } m_s < 0 \\ P_{\text{flip}} = \dfrac{\exp(-\beta m_s)}{\exp(\beta m_s)} \exp(-2\beta J s) & \text{if } m_s > 0 \end{cases}$$

where $m_s = \sum_{i \in C} h_i \sigma_i$. As explained in [16] this algorithm satisfies the conditions of ergodicity and detailed balance for the random field model. A Monte Carlo sweep is obtained when we attempt to flip a number of clusters like the volume of the system.

We believe that for models such as the RFIM, this kind of dynamic is capable of strongly reducing the problem related to the dynamic slowing down as it approaches the critical temperature. Using a single spin-flip dynamic a new configuration is obtained when we try to flip all the spins in the lattice. The probability of flipping a spin depends on the local fields through a term proportional to $p_{\text{flip}}(h) = \exp(-2\beta h_i)$. It is possible that some spins are aligned to a large local field; in this case such spins are almost impossible to be flipped. Sometimes if we flip this 'pinned' spin, the configuration obtained can be more probable. When one of these spins is taken as a part of a cluster the effect of the field of such a site is rounded by the other fields in the cluster. In this case the procedure that realizes the Monte Carlo dynamic is much more complicated than that of the Metropolis algorithm. Moreover, this kind of dynamic depends largely on the contest. It is therefore necessary to spend a lot of time optimizing the algorithm. Given n the number of spins in a cluster, m_s is a sum over n of independent variables with variance σ^2; hence m_s is a variable with variance $n\sigma^2$. For large clusters typical values of P_{flip} are infinitesimal. For this reason $P(R)$ should give more weight to small R. Nevertheless we need some large cluster in order to avoid pinning problems. A bad choice of $P(R)$ can cause equilibration times to be longer than that of the single-spin-flip dynamic so that the cluster algorithm becomes unusable.

4. Numerical results

Rieger and Young [14, 15] have carried out the most extensive Monte Carlo simulations in three dimensions in order to test the scaling relation validity. Using finite size scaling techniques they calculate all the critical exponents both with a Gaussian and with a bimodal probability distribution.

Making use of the cluster algorithm described in section 3 we carried out the Monte Carlo simulation in order to search for numerical evidence of the existence of a 'spin-glass'-like phase transition in the region above the critical temperature for a four-dimensional lattice. To this end, at each Monte Carlo sweep, for each disorder realization we measured the average magnetization $\langle m \rangle$, its square $\langle m^2 \rangle$, the average overlap $\langle q \rangle$, and its square $\langle q^2 \rangle$; with $m = 1/N \sum_{i=1}^{N} \sigma_i$ and q given in equation (5). In this way we calculate the four quantities given in (6). The two replica are such that they have the same realization of the disorder. They approach the equilibrium following two different Markovian processes, so in this case they can be considered independent. We performed the calculation using a four-dimensional lattice of size $L = 16$ with periodic boundary conditions. We measured the quantities in (6) for 13 different values of the temperature for each realization of the disorder. Starting from a high temperature region we cooled the system until it reaches 10% of the critical temperature. The hardest region to simulate is the one near the critical temperature. For these temperatures the system takes a great deal of time to reach equilibrium. Near T_c up to 250 Monte Carlo sweeps were needed to balance the system. At temperatures T sufficiently greater than T_c the system balances very fast. In contrast, near T_c the time

needed to balance the system become much too long. For this reason if we take the same amount of Monte Carlo sweeps at each temperature we would waste time. We used a range of temperature varying from 8.33 to 5.97 that correspond to $0.12 \leqslant \beta \leqslant 0.1675$ where $\beta = 1/T$.

Starting from $\beta = 0.12$ we cooled the system through the following rules

$$\beta = 0.12 + (k-1)0.005 \qquad k = 1 \ldots 8 \tag{7}$$

$$\beta = 0.1575 + (k-1)0.0025 \qquad k = 9 \ldots 13. \tag{8}$$

In this way we have more points near T_c. At each temperature we have used a Monte Carlo sweep number ($\#MCs$) given by

$$\#MCs = 64k^2 \qquad k = 1 \ldots 13.$$

A third of these were used to balance the system and the rest to take the measurements. For $k = 13$, i.e. near T_c, from the 10816 $\#MCs$, the first 3605 were used to reach the equilibrium. This is an order of amplitude greater than the calculated equilibration time. In this way, in the hardest region to simulate the thermal average there were performed more than 500 uncorrelated measurements. The average performed over the disorder were 850 samples. We need such a large number of samples because the quantities in (6) are highly non-self-averaging. For this reason the error caused by the disordered average is an order of amplitude greater than that given by the thermal average. Though the quantities in (6) are non-self-averaging it can be shown that the averages over the disorder can be used to analyse the critical behaviour of the system; in fact if we call $\overline{\log \chi}$ the average over the disorder of the logarithm of the susceptibility we can compare $\exp(\overline{\log \chi})$ with $\overline{\chi}$. It turns out that $\exp(\overline{\log \chi}) \sim \overline{\chi}$; in particular, for the disconnected magnetic susceptibility and the disconnected overlap susceptibility we have $\overline{\chi_{\rm dis}(m)}/[\exp(\overline{\log \chi_{\rm dis}(m)})] \sim 3$ and $\overline{\chi_{\rm dis}(q)}/[\exp(\overline{\log \chi_{\rm dis}(q)})] \sim 1$. These two ratios are constant for all the range of temperatures given in (7) and (8) so that we expect our numerical results to give the right critical behaviour.

The random field was chosen so that $h_R = 0.35T$ for different temperatures. As was pointed out in [14] for greater values of h_R/T the system is too difficult to balance and when the ratio is too small the system degenerates in the ferromagnetic model.

The quantities (6), near T_c are well fitted by some power law of the temperature. Both for the magnetic susceptibility and the overlap susceptibility we are sufficiently far from T_c that we can neglect the finite size effect. For the connected and disconnected magnetic susceptibility we use the following power law

$$\overline{\chi(m)} = C(T - T_c)^{-\gamma}$$

$$\overline{\chi(m)_{\rm dis}} = C_1(T - T_c)^{-\overline{\gamma}}. \tag{9}$$

In figure 1 $1/\chi(m)_{\rm con}$ and $1/\sqrt{\chi(m)_{\rm dis}}$ are plotted against the temperature. It is clear from the figure that the two exponents γ and $\overline{\gamma}$ are respectively slightly lower than one and two. The results of the least-square fit are such that

$$T_c = 5.72 \pm 0.04 \qquad \gamma = 0.94 \pm 0.02 \qquad \overline{\gamma} = 1.91 \pm 0.08. \tag{10}$$

The jacknife technique is used for the errors; the data are correlated since the same set of random fields is used at each temperature. According to previous simulations [15] the Schwartz inequality [7] seems to be valid as an equality. Within the error bars we have $2\gamma = \overline{\gamma}$, that is

$$\overline{\eta} = 2\eta.$$

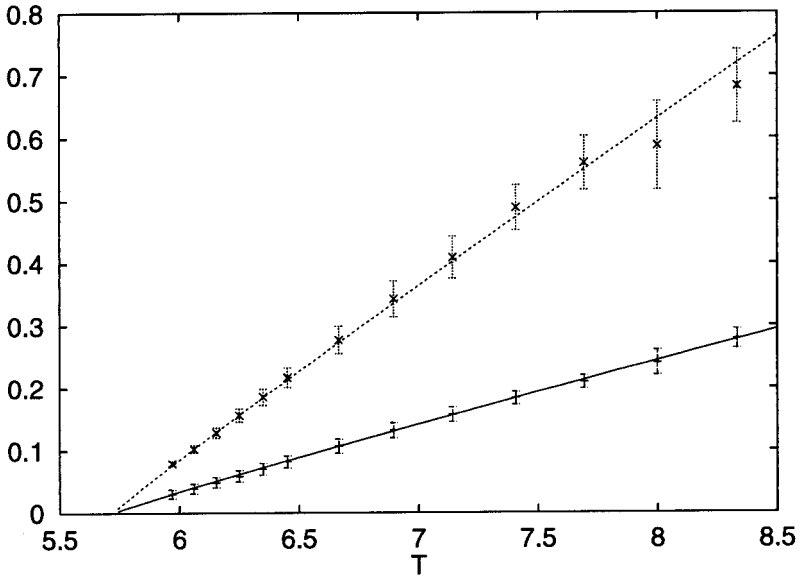

Figure 1. The full curve and the broken curve represent the least-squares fit results. The full curve is the plot of $1/f(x)$ where $f(x) = 8.9(T - T_c)^{-0.94}$ and the broken curve is the plot of $1/f_1(x)^{(-1/2)}$ where $f_1(x) = 12.1(T - T_c)^{-1.91}$. The intersection with the abscissa gives the critical temperature.

Considering that $\overline{\langle q^2 \rangle} \to 1$ when $T \to \infty$ we expect the connected overlap susceptibility to have a power behaviour given by

$$\overline{\chi(q)_{\rm con}} = B(T - T_{\rm b})^{-\omega} + D. \tag{11}$$

Because of the presence of the random field the disconnected overlap susceptibility has a non-vanishing term. We expect a power law behaviour given by

$$\overline{\chi(q)_{\rm dis}} = B_1(T - T_{\rm b})^{-\overline{\omega}} + D_1. \tag{12}$$

We report in figure 2 the least-squares fit result. As for the results in (10) the errors are calculated with the jackknife technique. We find

$$T_{\rm b} = 5.88 \pm 0.04 \qquad \omega = 0.6 \pm 0.1 \qquad \overline{\omega} = 0.42 \pm 0.05. \tag{13}$$

Equation (12) is valid near the critical temperature. It can be argued that the results found for the two critical temperatures, may be an artefact of the power-law behaviour used in (12). There could be a temperature drift in the non-singular term as the temperature is not too far from T_c. To take care of this effect we add in (12) a linear term in the temperature vanishing near T_c. We use a temperature dependence given by

$$\overline{\chi(q)_{\rm dis}} = B_1(T - T^*)^{-\overline{\omega}} + D_1[1 + D_0(T - T^*)/T^*]. \tag{14}$$

If we fix T^* we can perform a four parameters fit. When $T^* = T_c$ the values of χ^2 obtained are greater than that are obtained in the previous fit. If we set $T^* = T_b$ we recover the results obtained using (12). In this case the temperature can be neglected and the results are well represented by the power law given in (12). The least-squares fit results are reported in table 1.

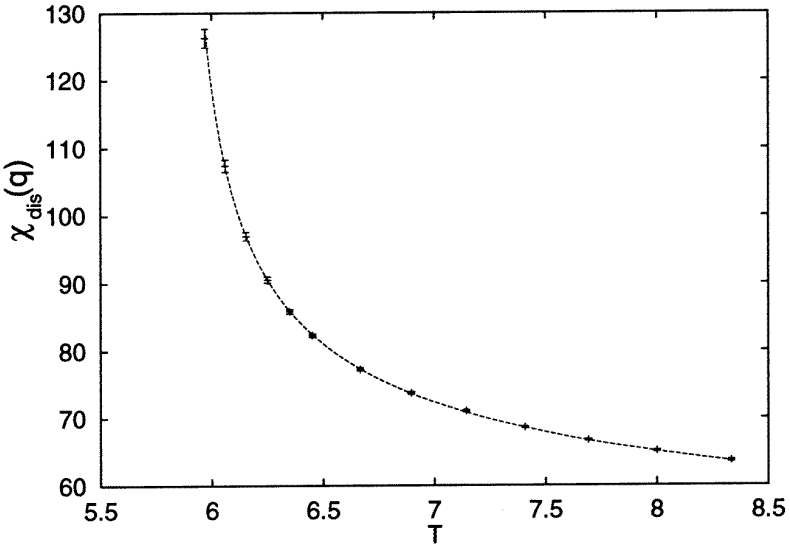

Figure 2. Disconnected overlap susceptibility. The full curve is the least-squares fit result $f(x) = 33(T - 5.88)^{-0.42} + 41$.

Table 1. In the first column we have the least-squares fit results obtained using the power law given in (12). The results in column two and three are obtained using the temperature dependence given in (14). In the last row we have the values of χ^2 calculated during the fit. The values in column one and three are almost equal. When $T^* = T_b$ we have $D_0 \sim 0$ so that we can assume that (12) is a good approximation for all the temperatures used.

	(12)	$T^* = T_c$	$T^* = T_b$
B_1	33 ± 2	13.3 ± 0.9	32 ± 2
$\overline{\gamma}$	0.42 ± 0.02	1.09 ± 0.04	0.43 ± 0.02
D_1	41 ± 2	65 ± 1	43 ± 2
D_0	0	-0.22 ± 0.02	-0.03 ± 0.02
χ^2	2.14	4.76	1.98

5. Conclusion

From data analysis the overlap susceptibility and the magnetic susceptibility seem to diverge at two different points. It turns out that $T_b > T_c$. If this is the case the three transition scheme, obtained through RSB techniques, should be correct. A more extensive Monte Carlo simulation should be done near T_c, using finite size scaling techniques, in order to confirm the result obtained. It should be interesting as well to study the overlap distribution probability in the region under T_b. Another result is given from the comparison between γ and $\overline{\gamma}$. We find $\overline{\gamma} = 2\gamma$, according to this result we have found one more scaling relation so that the independent critical exponents are two instead of three. In four dimensions also, the dimensional reduction does not give the correct result, in fact $\gamma = 0.94$ is very far from $7/4$ which is a prediction of the dimensional reduction.

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